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## Theory of threshold characteristics of quantum dot lasers

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**Abstract.** A theory of the gain and threshold current of a semiconductor quantum dot (QD) laser has been developed which takes account of the line broadening caused by fluctuations in QD size. Expressions for the threshold current versus the surface density of QDs, QD size dispersion and total losses have been obtained in explicit form. Optimization of the structure has been carried out, aimed at minimizing the threshold current density. The characteristic temperature of QD laser has been calculated considering carrier recombination in the optical confinement layer and violation of the charge neutrality in QDs.

Quantum dot (QD) lasers are of particular interest because of the following expected advantages over the conventional quantum well lasers: the narrower gain spectra, significantly lower threshold currents and the weaker temperature dependence of the latter [1]. As a consequence of quantum confinement in all the three dimensions, the energy spectra of carriers in QDs are discrete. For this reason, structures with QDs have generated much interest as a new class of artificially structured materials with tunable (through varying the composition and size) energies of discrete atomic-like states that are ideal for use in laser structures.

Here, we briefly review the theory of the threshold current of a QD laser, developed in [2–6].

Equilibrium or nonequilibrium filling of carrier levels in QDs has been shown to be realized depending on temperature  $T$ , QD sizes and conduction and valence band offsets at the QD-optical confinement layer (OCL) heteroboundary  $\Delta E_{c,v}$  [2, 3].

If the characteristic times of thermally excited escapes of an electron and hole from a QD are small compared with the radiative lifetime in QDs,  $\tau_{\text{QD}}$ , redistribution of carriers from one QD to another occurs, and quasi-equilibrium distributions are established with the corresponding quasi-Fermi levels. As a consequence of such a redistribution, the level occupancies (and numbers of carriers) in various QDs will differ.

The condition for the equilibrium filling of QDs may be written as  $T > T_g$  where

$$T_g = \max \left( \frac{\Delta E_c - \varepsilon_n}{\ln(\sigma_n v_n N_c \tau_{\text{QD}})}, \frac{\Delta E_v - \varepsilon_p}{\ln(\sigma_p v_p N_v \tau_{\text{QD}})} \right). \quad (1)$$

Here  $\varepsilon_{n,p}$  are the quantized energy levels of an electron and hole in a mean-sized QD (measured from the corresponding band edges),  $\sigma_{n,p}$  the cross sections of electron and hole capture into a QD,  $v_{n,p}$  the thermal velocities, and  $N_{c,v}^{\text{OCL}}$  the conduction- and valence-band effective densities of states for the OCL material.

The peak modal gain appearing in the threshold condition is

$$g = \frac{\xi}{4} \left( \frac{\lambda_0}{\sqrt{\epsilon}} \right)^2 \frac{1}{\tau_{\text{QD}}} \frac{\hbar}{(\Delta \epsilon)_{\text{inhom}}} \frac{\Gamma}{a} N_S (f_n + f_p - 1) \quad (2)$$

where  $f_{n,p}$  are the electron and hole level occupancies averaged over the QD ensemble,  $\xi$  a numerical constant appearing in QD-size distribution function ( $\xi = 1/\pi$  and  $\xi = 1/\sqrt{2\pi}$  for the Lorentzian and Gaussian functions, respectively),  $\lambda_0$  the wavelength at the maximum gain,  $a$  the mean size of QDs,  $\Gamma$  the optical confinement factor in a QD layer (along the transverse direction in the waveguide),  $N_S$  the surface density of QDs and  $(\Delta\varepsilon)_{\text{inhom}}$  the inhomogeneous line broadening due to the QD-size dispersion.

The current density is

$$j = \frac{eN_S}{\tau_{\text{QD}}} f_n f_p + ebB n_1 p_1 \frac{f_n f_p}{(1-f_n)(1-f_p)} \quad (3)$$

where  $b$  is the OCL thickness, and  $B$  is the radiative constant for the OCL,

$$n_1 = N_c^{\text{OCL}} \exp\left(-\frac{\Delta E_c - \varepsilon_n}{T}\right) \quad p_1 = N_v^{\text{OCL}} \exp\left(-\frac{\Delta E_v - \varepsilon_p}{T}\right). \quad (4)$$

If the characteristic times of thermally excited escapes of carriers from a QD are large compared with the radiative lifetime in QDs (relatively low temperatures,  $T < T_g$ ), the redistribution of carriers from one QD to another and establishment of quasi-Fermi levels for the conduction and valence bands do not occur; in this case, nonequilibrium filling of QDs is realized. Having no time to leave a QD, the carriers recombine there. Since the initial numbers of carriers injected into various QDs are the same, the QD level occupancies are also the same. The contribution of each QD to the lasing is the same. In this case, too, the peak modal gain is given by (2) wherein the level occupancies common to all QDs appear. The current density is given by

$$j = \frac{eN_S}{\tau_{\text{QD}}} f_n f_p + \frac{ebB}{\sigma_n \sigma_p v_n v_p \tau_{\text{QD}}^2} \frac{f_n^2 f_p^2}{(1-f_n)(1-f_p)}. \quad (5)$$

In (3) and (5), the first and second terms are the current densities associated with the spontaneous radiative recombination in QDs and in the OCL, respectively.

With (2) and the threshold condition ( $g = \beta$  where  $\beta$  is the total loss coefficient), the population inversion in QDs required for lasing may be written as

$$f_n + f_p - 1 = \frac{N_S^{\min}}{N_S} \quad (6)$$

where  $N_S^{\min}$  is the minimum tolerable surface density of QDs required to attain lasing at given loss  $\beta$  and inhomogeneous line broadening  $(\Delta\varepsilon)_{\text{inhom}}$  [2]–[4, 6]:

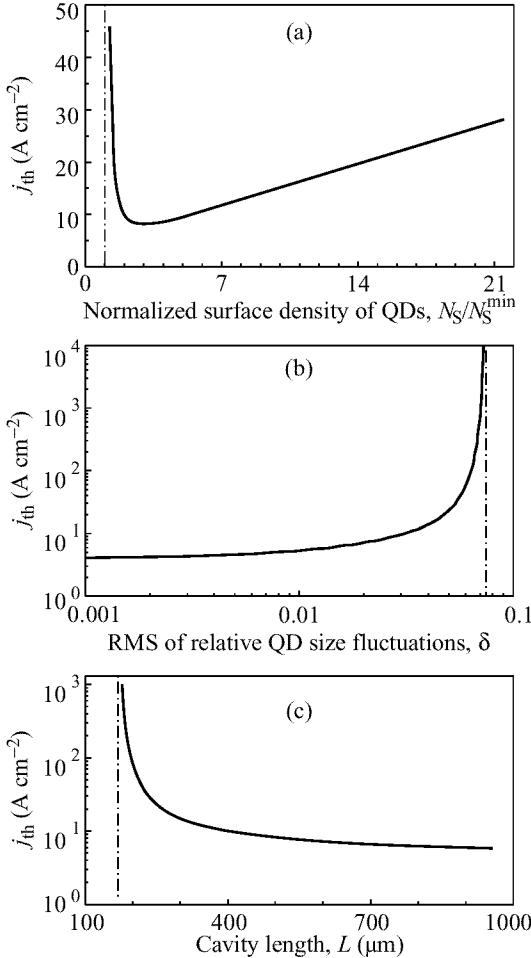
$$N_S^{\min} = \frac{4}{\xi} \left(\frac{\sqrt{\epsilon}}{\lambda_0}\right)^2 \tau_{\text{QD}} \frac{(\Delta\varepsilon)_{\text{inhom}}}{\hbar} \beta \frac{a}{\Gamma}. \quad (7)$$

The mean level occupancies in QDs are related to each other by (6). The second equation relating  $f_n$  to  $f_p$  should be derived from the solution of the corresponding self-consistent problem for the electrostatic field distribution across the junction and depends on the QD laser design [4].

The dependence of  $j_{\text{th}}$  on  $N_S$  is nonmonotonic (Fig. 1(a)). In the case of equilibrium filling of QDs, whatever the specific type of the second equation relating  $f_n$  to  $f_p$  is, the

minimum threshold current density has been shown to be [3, 4]

$$j_{\text{th}}^{\min} = \left[ \left( \frac{eN_S^{\min}}{\tau_{\text{QD}}} \right)^{1/2} + (ebBn_1p_1)^{1/2} \right]^2. \quad (8)$$



**Fig. 1.** Threshold current density versus (a) the normalized surface density of QDs, (b) RMS of relative QD size fluctuations and (c) cavity length.

In the special case of a symmetric structure ( $f_n = f_p$ ),

$$f_{n,p} = \frac{1}{2} \left( 1 + \frac{N_S^{\min}}{N_S} \right). \quad (9)$$

The electron and hole level occupancies at the lasing threshold may be expressed as a function of the root mean square (RMS) of relative QD size fluctuations  $\delta$  or the cavity

length  $L$  as follows:

$$f_{n,p} = \frac{1}{2} \left( 1 + \frac{\delta}{\delta^{\max}} \right) \quad f_{n,p} = \frac{1}{2} \left( 1 + \frac{L^{\min}}{L} \right). \quad (10)$$

All other parameters of the structure being constant,  $\delta^{\max}$  and  $L^{\min}$  are the maximum tolerable RMS of relative QD size fluctuations and the minimum tolerable cavity length at which lasing is possible. For such  $\delta$  or  $L$ , the surface density of QDs is equal to its minimum tolerable value  $N_S^{\min}$ .

As  $N_S \rightarrow N_S^{\min}$ , or  $\delta \rightarrow \delta^{\max}$ , or  $L \rightarrow L^{\min}$ , the mean electron and hole level occupancies in QDs tend to unity ( $f_{n,p} \rightarrow 1$ ), which demands infinitely high free-carrier densities in the OCL. As a result,  $j_{\text{th}}$  increases infinitely (Figs. 1(a)–1(c)).

As  $\delta \rightarrow 0$ , or  $L \rightarrow \infty$  ( $\beta \rightarrow 0$ ),  $j_{\text{th}}$  decreases and approaches the transparency current density (Figs. 1(b) and 1(c)).

Ideally, the  $j_{\text{th}}$  of a QD laser must be temperature-independent and the characteristic temperature,  $T_0 = (\partial \ln j_{\text{th}} / \partial T)^{-1}$ , must be infinitely high [1]. This would be so indeed if the overall injection current went entirely into the radiative recombination in QDs and the charge neutrality in QDs were the case [5, 6]. In fact, because of the presence of free carriers in the OCL, a fraction of the injection current is wasted therein. This fraction goes into the recombination processes in the OCL (the second term in (3) and (5)).

In the case of nonequilibrium filling of QDs ( $T < T_g$ ), the threshold current is essentially temperature-independent. More precisely, there is a weak temperature dependence of  $j_{\text{th}}$  due to the temperature dependence of the cross sections of carrier capture into a QD  $\sigma_{n,p}$ , thermal velocities  $v_{n,p}$  and radiative constant  $B$  (see (5)).

In the case of equilibrium filling of QDs ( $T > T_g$ ), the current component associated with the recombination in the OCL (second term in (3)),  $j_{\text{OCL}}$ , depends on  $T$  exponentially. As a result,  $j_{\text{th}}$  must become temperature dependent, especially at high  $T$ . Hence  $T_0$  must become finite.

If the charge neutrality in QDs were the case ( $f_n = f_p$ ),  $f_{n,p}$  and hence the current component associated with the recombination in QDs (the first term in (3)),  $j_{\text{QD}}$ , would be temperature-independent. Examination of the problem shows [4]–[6] that the electron and hole level occupancies in QDs at the lasing threshold,  $f_n$  and  $f_p$ , become temperature-dependent if the violation of the charge neutrality in QDs is taken into account properly. Thus, correct consideration of the QD charge reveals the  $T$ -dependence of  $j_{\text{QD}}$ .

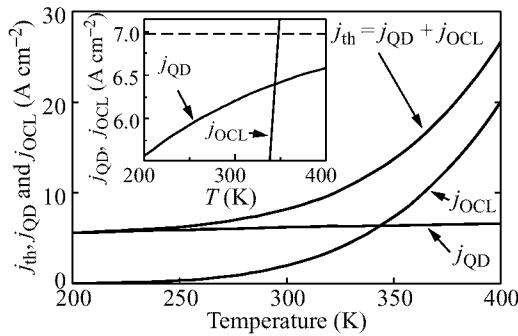
The characteristic temperature of a QD laser,  $T_0$ , can be represented as [5, 6]

$$\frac{1}{T_0} = \frac{j_{\text{QD}}}{j_{\text{QD}} + j_{\text{OCL}}} \frac{1}{T_0^{\text{QD}}} + \frac{j_{\text{OCL}}}{j_{\text{QD}} + j_{\text{OCL}}} \frac{1}{T_0^{\text{OCL}}} \quad (11)$$

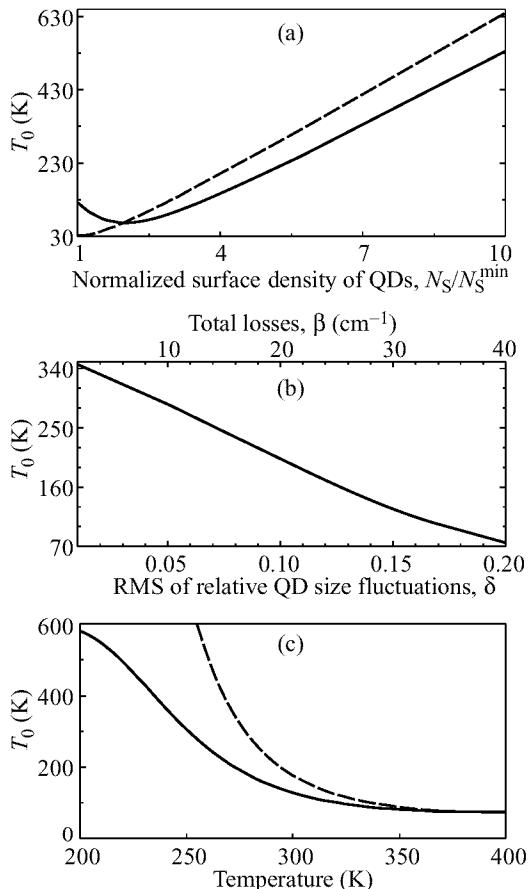
where  $T_0^{\text{QD}}$  and  $T_0^{\text{OCL}}$  are defined similarly to  $T_0$  for the functions  $j_{\text{QD}}(T)$  and  $j_{\text{OCL}}(T)$ , respectively:  $1/T_0^{\text{QD}} = \partial \ln j_{\text{QD}} / \partial T$  and  $1/T_0^{\text{OCL}} = \partial \ln j_{\text{OCL}} / \partial T$ .

Hence, the reciprocal of  $T_0$  is a sum of the reciprocals of  $T_0^{\text{QD}}$  and  $T_0^{\text{OCL}}$ , each weighted by the relative contribution of the respective component of  $j_{\text{th}}$ .

The  $T$ -dependences of  $f_{n,p}$  are much weaker compared to that of the exponential in (4). Consequently,  $j_{\text{QD}}$  increases with  $T$  much more slowly than  $j_{\text{OCL}}$  does (Fig. 2). Hence,  $T_0^{\text{QD}}$  is much greater than  $T_0^{\text{OCL}}$ . Nevertheless, as it can be seen from (11),  $1/T_0$  is controlled not only by  $1/T_0^{\text{QD}}$  and  $1/T_0^{\text{OCL}}$ , but by the relative contributions of the threshold current density components,  $j_{\text{QD}}/j_{\text{th}}$  and  $j_{\text{OCL}}/j_{\text{th}}$ , as well. For this reason, under temperature



**Fig. 2.** Threshold current density and its components versus the temperature for  $N_S = 7.7 \times 10^{10} \text{ cm}^{-2}$ . The inset shows  $j_{\text{QD}}(T)$  and  $j_{\text{OCL}}(T)$  on an enlarged (along the vertical axis) scale. The broken line depicts  $j_{\text{QD}}$  calculated assuming the charge neutrality in QDs.



**Fig. 3.** Characteristic temperature  $T_0$  versus (a) the normalized surface density of QDs, (b) RMS of relative QD size fluctuations  $\delta$  (for  $\beta = 10 \text{ cm}^{-1}$ , bottom axis) and the total loss  $\beta$  (for  $\delta = 0.05$ , top axis) for  $N_S = 1.3 \times 10^{11} \text{ cm}^{-2}$  and (c) temperature for  $N_S = 7.7 \times 10^{10} \text{ cm}^{-2}$ . The broken curves depict  $T_0$  calculated assuming the charge neutrality in QDs.

conditions wherein  $j_{\text{th}}$  is controlled by  $j_{\text{QD}}$  (Fig. 2), the contribution of the first term in the right-hand side of (11) is every bit as important as that of the second term.

For  $N_S$  fairly greater than  $N_S^{\min}$ ,  $T_0$  increases with  $N_S$  (Fig. 3(a)). The point is that the less temperature-sensitive component of  $j_{\text{th}}$ , i.e.,  $j_{\text{QD}}$ , increases with  $N_S$ , whereas the more temperature-sensitive component of  $j_{\text{th}}$ , i.e.,  $j_{\text{OCL}}$ , decreases.

The greater the RMS of relative QD size fluctuations  $\delta$  or the total loss  $\beta$  (i.e., the less perfect the structure), the lower  $T_0$  at given  $T$  and given other parameters (Fig. 3(b)).

The characteristic temperature depends strongly on  $T$ ;  $T_0$  falls off profoundly with increasing  $T$  (Fig. 3(c)). A drastic decrease in  $T_0$  occurs in passing from the temperature conditions wherein  $j_{\text{th}}$  is controlled by radiative recombination in QDs (Fig. 2) to those under which  $j_{\text{th}}$  is controlled by radiative recombination in the OCL (Fig. 2). The  $T_0$  values at  $T = 200$  and  $300$  K are  $582$  and  $128$  K, respectively (Fig. 3(c)).

We emphasize that the tendency for  $T_0$  to decrease drastically with  $T$  seems to be in the line with the available experimental results [7].

As Fig. 3(c) suggests, at relatively low  $T$  (when  $j_{\text{th}}$  is controlled by  $j_{\text{QD}}$ ), the actual  $T_0$  differs significantly from that calculated assuming the charge neutrality in QDs.

As our example, we use a GaInAsP/InP heterojunction structure lasing at  $1.55 \mu\text{m}$  [3]–[6]. A device with OCL thickness of  $b = 0.28 \mu\text{m}$  and an as-cleaved facet at both ends is considered. A Gaussian distribution of the relative QD size fluctuations is assumed. The mean size of cubic QDs is taken to be  $150 \text{ \AA}$ . The surface density of QDs, RMS of relative QD size fluctuations, cavity length, and temperature are taken to be  $N_S = 6.1 \times 10^{10} \text{ cm}^{-2}$ ,  $\delta = 0.025$  (5%),  $L = 500 \mu\text{m}$ , and  $T = 300$  K, respectively, unless otherwise specified. The corresponding values of the minimum tolerable surface density of QDs, maximum tolerable RMS of relative QD size fluctuations, and minimum tolerable cavity length required to attain lasing are  $N_S^{\min} = 2.1 \times 10^{10} \text{ cm}^{-2}$ ,  $\delta^{\max} = 0.074$  (14.8%), and  $L^{\min} = 170 \mu\text{m}$ , respectively.

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